

CERN-TH/97-298
NORDITA-97/78P
hep-ph/9710538
December 1997

HIGH TEMPERATURE DIMENSIONAL REDUCTION AND PARITY VIOLATION

K. Kajantie^{a,b1}, M. Laine^{a2}, K. Rummukainen^{c3} and M. Shaposhnikov^{a4}

^a*Theory Division, CERN, CH-1211 Geneva 23, Switzerland*

^b*Department of Physics, P.O.Box 9, 00014 University of Helsinki, Finland*

^c*NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*

Abstract

The effective super-renormalizable 3-dimensional Lagrangian, describing the high temperature limit of chiral gauge theories, has more symmetry than the original 4d Lagrangian: parity violation is absent. Parity violation appears in the 3d theory only through higher-dimensional operators. We compute the coefficients of dominant P-odd operators in the Standard Electroweak theory and discuss their implications. We also clarify the parametric accuracy obtained with dimensional reduction.

¹keijo.kajantie@cern.ch

²mikko.laine@cern.ch

³kari@nordita.dk

⁴mshaposh@nxth04.cern.ch

1 Introduction

A general feature of effective field theories is that they may have more symmetries than the corresponding original theories. These extra symmetries are then broken by higher-dimensional operators in the effective theory. In many cases, the higher order operators are non-renormalizable whereas the “effective” Lagrangian is renormalizable. A familiar example is GUT-induced baryon number violation [1] which is absent in the renormalizable Standard Model Lagrangian. Another classic example is the four-Fermion interaction, which introduces, e.g., strangeness non-conservation into QED+QCD. Additional symmetries may also appear in the case of non-renormalizable effective theories, like in chiral effective Lagrangians for QCD [2].

The purpose of this paper is to point out that a similar situation may arise in dimensionally reduced effective field theories describing the high temperature thermodynamics of various weakly coupled gauge theories [3–11] (for a review, see [12]). In this context, the effective theory is 3-dimensional and super-renormalizable. It does not contain parity (P) violation, nor in many cases, such as the Standard Model, charge conjugation (C) violation. However, if the original 4d theory breaks P or C, it will induce P or C breaking higher dimensional operators into the effective theory.

To be concrete, consider the electroweak sector the Standard Model (SM), disregarding the U(1) group, at non-zero temperature T but with zero chemical potentials $\mu = 0$ for all conserved charges. The super-renormalizable effective action relevant for this case is⁵

$$S = \int d^3x \left\{ \frac{1}{2} \text{Tr} F_{ij} F_{ij} + (D_i \phi)^\dagger (D_i \phi) + m_3^2 \phi^\dagger \phi + \lambda_3 (\phi^\dagger \phi)^2 \right. \\ \left. + \text{Tr} [D_i, A_0] [D_i, A_0] + m_D^2 \text{Tr} A_0 A_0 + \lambda_A (\text{Tr} A_0 A_0)^2 + 2h_3 \phi^\dagger \phi \text{Tr} A_0 A_0 \right\}, \quad (1)$$

where $F_{ij}^a = \partial_i A_j^a - \partial_j A_i^a + g_3 \epsilon^{abc} A_i^b A_j^c$, $F_{ij} = T^a F_{ij}^a$, $D_i \phi = (\partial_i - ig_3 T^a A_i^a) \phi$, $A_0 = T^a A_0^a$, and $T^a = \tau^a/2$. The τ^a are the Pauli matrices. The factor $1/T$ multiplying the action has been scaled into the fields and the coupling constants, so that the fields have the dimension $\text{GeV}^{1/2}$ and the couplings g_3^2 , λ_3 have the dimension GeV . The connection between the couplings in eq. (1) and the physical parameters of the 4d theory is given explicitly in [6]. The P and C violating chiral fermions contribute only through the parametric dependence of the couplings on the number of families N_f and on the top quark Yukawa coupling g_Y . The terms of lowest dimensionality neglected in eq. (1) have $\text{dim}=6$ in 4d units; an example is $(\phi^\dagger \phi)^3$. The P and C conserving $\text{dim}=6$ operators arising in dimensional reduction have been studied in [7, 14, 15], and P breaking operators have been discussed in [16–20].

It is now obvious that the P and C violating effects of the 4d theory must appear in the higher dimensional operators of the effective theory. In fact, one expects that

⁵The super-renormalizable effective 3d theory for the case of non-zero chemical potentials was discussed in [13].

the dominant effects will come from the integration out of the top quark. The leading diagrams, beyond those already included in the computation of the coupling constant relations for the theory in eq. (1), are shown in Fig. 1. The corresponding operator will be given in eq. (11) below.

2 P and C breaking operators

Before computing the diagram in Fig. 1 it is illuminating to analyse the situation more generally. To this end we first specify the transformation properties under discrete 4d CPT transformations of the fields appearing in the 3d action $S = S[A_i, A_0, \phi]$, then find out all possible $J^{PC} = 0^{+-}, 0^{-+}, 0^{--}$ operators of lowest dimensionalities and finally see which of these are really induced in the transition from the 4d to the 3d effective theory and with what coefficients. Since the CP violation of the SM is very small, one expects the coefficients of any induced $0^{+-}, 0^{-+}$ operators to be tiny so that the main interest is in the P and C violating operators with $J^{PC} = 0^{--}$. Note that in some extensions of the Standard Model, such as in the MSSM, CP-violation can appear in the bosonic sector even in the super-renormalizable vertices of the 3d theory, and could thus have correspondingly larger effects.

The transformation properties of the objects appearing in the effective theory $S = S[A_i, A_0, \phi]$ under the 4d discrete symmetries C, P and T are shown in Table 1. With these one can explicitly verify that the terms in eq. (1) are separately invariant under C, P and T.

Consider now the possible P or C violating locally gauge invariant operators in order of increasing dimensionality.

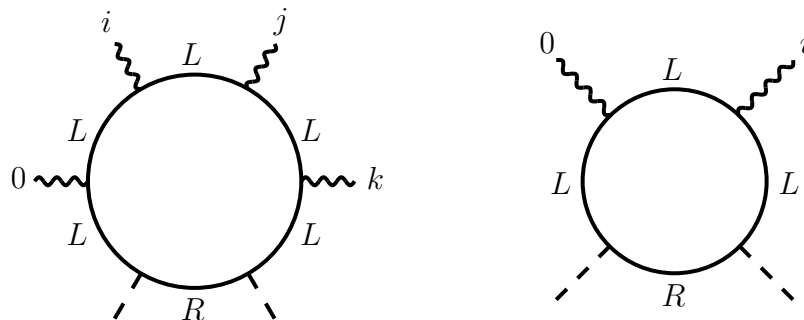


Figure 1: The graphs giving the dim=6 operator O_4^{--} in eq. (11). The solid line is a (top) quark propagator, the dashed line a Higgs field and the wiggly line a gauge field. The symbols L and R indicate the handednesses of the quarks. For the dim=6 operator one needs the momentum dependence of the 4-leg diagram, which supplies two more indices j, k . An obvious 5-leg diagram is not shown.

1. Dim=5. The lowest possible dimensionality is dim=5 in 4d units (after rescaling to 3d, the dimension is $3\frac{1}{2}$ or 4). It is helpful to write the spatial gauge fields in terms of

$$B_i = \frac{1}{2}\epsilon_{ijk}F_{jk}. \quad (2)$$

The P violating locally gauge invariant operators O^{PC} that can in principle emerge, are then

$$O_1^{-+} = ic_1\phi^\dagger\{D_i, B_i\}\phi, \quad (3)$$

$$O_2^{-+} = ic_2\text{Tr}[D_i, A_0][B_i, A_0], \quad (4)$$

$$O_3^{-+} = c_3\epsilon_{ijk}\text{Tr} B_i[D_j, B_k]. \quad (5)$$

Here the number of possible operators has been reduced by making use of the Bianchi identity $[D_i, B_i] = 0$, of the antisymmetry in permutations of the trace $\text{Tr} ABC$ for $\text{SU}(2)$, and of the property $\text{Tr} A[D, B] = -\text{Tr} B[D, A]$. The coefficients c_i are real in order to make the operators Hermitian (or, as scalars, real) in Minkowski space. The operators in eqs. (3)–(5) clearly violate P, but using Table 1, one can see that they conserve C. Thus they are CP-violating. Moreover, they conserve T, thus violating CPT (of the original 4d theory). Terms of this kind can only arise in connection with non-zero chemical potentials [16] which we assumed are zero.

One can also write down P-conserving but C-violating operators, $J^{PC} = 0^{+-}$:

$$i\phi^\dagger A_0\phi\phi^\dagger\phi, \quad i\phi^\dagger A_0\phi\text{Tr} A_0A_0, \quad (6)$$

	C	P	T	CPT
ϕ	ϕ^*	ϕ	ϕ	ϕ^*
A_0	$-A_0^*$	A_0	A_0	$-A_0^*$
A_i	$-A_i^*$	$-A_i$	$-A_i$	$-A_i^*$
i	i	i	$-i$	$-i$
D_0	D_0^*	D_0	$-D_0$	$-D_0^*$
D_i	D_i^*	$-D_i$	D_i	$-D_i^*$
B_i	$-B_i^*$	B_i	$-B_i$	B_i^*
F_{ij}	$-F_{ij}^*$	F_{ij}	$-F_{ij}$	F_{ij}^*

Table 1: The transformation properties of the bosonic fields with Minkowskian A_0 . Minkowski space is used here because it is needed for seeing the Hermiticity properties of the operators. In the text the operators are written with Euclidian A_0 , $A_0 \equiv A_0^E = -iA_0^M$, so that they can consistently be added to the Euclidian action in eq. (1). Note that apart from for ϕ , C for $\text{SU}(2)$ corresponds to the global gauge transformation $g = i\tau^2$, and thus there is no C-violation without ϕ . The transformations could also be written in some other forms; for instance, the T-transformation properties of A_0^M , A_i are equivalent to $iA_0^M \rightarrow -(iA_0^M)^*$, $iA_i \rightarrow (iA_i)^*$.

$$i\phi^\dagger[D_i, [D_i, A_0]]\phi, \quad i\phi^\dagger\{D_i, \{D_i, A_0\}\}\phi. \quad (7)$$

When written in Minkowski-space using $A_0 \equiv A_0^E = -iA_0^M$, these are seen to conserve T and thus to violate CPT. Hence the operators in eqs. (6), (7) can again only arise in connection with non-zero chemical potentials. The same holds for $J^{PC} = 0^{++}$ operators odd in T, such as

$$\phi^\dagger\{D_i, [D_i, A_0]\}\phi. \quad (8)$$

Note that for zero chemical potentials there are no regular $J^{PC} = 0^{++}$ T-even operators induced by dimensional reduction at dim=5, either [15].

2. Dim=6. Consider then the next dimension, dim=6 in 4d units (dim=4, $4\frac{1}{2}$ or 5 in 3d units). One can find several CP-violating $J^{PC} = 0^{-+}$ operators, for example

$$i\partial_k(\phi^\dagger\phi)\text{Tr } A_0 B_k, \quad i\partial_k(\text{Tr } A_0 A_0)\text{Tr } A_0 B_k, \quad (9)$$

$$\epsilon_{ijk}\text{Tr } [D_i, A_0][D_j, A_0][D_k, A_0], \quad i\epsilon_{ijk}\text{Tr } [D_i, [D_j, F_{km}]] [D_m, A_0]. \quad (10)$$

When written in Minkowski-space, one can see that these operators are real, have $T=-1$, and hence have $CPT=+1$. Thus they can appear in the effective Lagrangian with very small coefficients coming from the CP-violation in the original theory. In the MSSM with new sources of CP-violation and more bosonic fields, many other kinds of CP-violating operators can arise, as well.

On the other hand, there are at dim=6 also the following two independent CP-even operators:

$$\begin{aligned} O_4^{--} &= c_4\phi^\dagger\{D_i, \{A_0, B_i\}\}\phi \\ &= 2ic_4\text{Im}[\phi^\dagger D_i\phi]\text{Tr } A_0 B_i, \end{aligned} \quad (11)$$

$$O_5^{--} = c_5\phi^\dagger[D_i, [A_0, B_i]]\phi. \quad (12)$$

These operators are even in T, thus again conserving CPT. The two forms of O_4^{--} in eq. (11) differ by a 3-divergence for SU(2), and the second form states explicitly that O_4^{--} is purely imaginary in Euclidian space for real c_4 . Since O_4^{--} , O_5^{--} conserve CP, the coefficients need not be vanishingly small.

The operators in eqs. (11),(12) can clearly come from the diagrams in Fig. 1. To compute the operator corresponding to these diagrams it is simplest to take the 6-leg diagram since then the external legs can be taken at zero momentum. If p is the loop momentum and the external vector legs have the indices $\mu\nu\alpha\beta$, then the γ_5 -part of the trace over the fermion loop has the structure

$$-2p_\gamma(p_\nu\epsilon_{\mu\gamma\alpha\beta} + p_\beta\epsilon_{\mu\nu\alpha\gamma}) + p^2\epsilon_{\mu\nu\alpha\beta}. \quad (13)$$

This is now summed over $p_0 = (2n+1)\pi T$ and integrated over \mathbf{p} by first using $p_0 p_i \rightarrow 0$, $p_i p_j \rightarrow \frac{1}{3}\mathbf{p}^2\delta_{ij}$. One then obtains the structure

$$\epsilon_{ijk}\phi^\dagger(A_0 A_i A_j A_k + A_i A_0 A_j A_k + A_i A_j A_0 A_k + A_i A_j A_k A_0)\phi. \quad (14)$$

Reintroducing D_i and F_{ij} gives precisely the structure in eq. (11), so that to 1-loop order,

$$c_4 = \frac{2}{3}g^2g_Y^2TD_6, \quad (15)$$

$$c_5 = 0, \quad (16)$$

where the extra T comes from going into 3d units and

$$D_6 = \oint_{p_f} \frac{1}{(p^2)^3} = T \sum_n \int \frac{d^3p}{(2\pi)^3} \frac{1}{[\mathbf{p}^2 + (2n+1)^2(\pi T)^2]^3} = \frac{7\zeta(3)}{128\pi^4 T^2}. \quad (17)$$

The structure in eq. (12) would have been obtained if the signs of the first and last terms in eq. (14) had been opposite.

We thus find that to leading order the inclusion of parity violating effects to the 3d effective theory of finite temperature SM leads to a purely imaginary higher dimensional term in the Euclidian action. The reason is actually simple: in Minkowski space the action corresponding to the diagrams in Fig. 1 must be real. Since the action is linear in A_0 , the transition to Euclidian space brings in one imaginary unit i .

The fact that the Euclidian P-violating operator O_4^{--} is imaginary, also implies that lattice Monte Carlo simulations of P-violating effects are in practice very difficult. Nevertheless, the new terms can have significant effects, as discussed below.

Note that both of the operators in eqs. (11), (12) contain the field A_0 . On the other hand, around the electroweak phase transition temperature, the field A_0 can be integrated out. This is because A_0 has a mass parameter which is always parametrically of order $m_D^2 \sim g^2 T^2$. In contrast, the mass parameter of the scalar field is $m_3^2 \sim g^4 T^2$ around the critical temperature, since there the 1-loop term $\sim g^2 T^2$ cancels against the tree-level term $\sim -m_H^2$. Thus near $T = T_c$, A_0 is “heavy” and ϕ, A_i are “light” and A_0 can be integrated out. Above the critical temperature, both A_0 and ϕ are “heavy” and can either both be kept in the effective theory or both be integrated out. In the theory without A_0 , the operators O_4^{--} , O_5^{--} do not exist and one has to go to still higher dimensions.

3. Dim=7. We are now interested in P and C violating but CP-even operators which do not contain A_0 and which could hence appear in the effective theory with only A_i, ϕ . The following operators exist at dim=7:

$$O_6^{--} = c_6 \phi^\dagger B_i \phi \partial_i \phi^\dagger \phi, \quad (18)$$

$$O_7^{--} = i c_7 \epsilon_{ijk} \phi^\dagger [D_i, [B_j, B_k]] \phi, \quad (19)$$

$$O_8^{--} = c_8 \phi^\dagger \{[D_i, B_j], \{D_i, D_j\}\} \phi, \quad (20)$$

$$O_9^{--} = c_9 \phi^\dagger [\{D_i, B_j\}, \{D_i, D_j\}] \phi. \quad (21)$$

There cannot be an O^{--} operator without ϕ , since for SU(2) C corresponds just to a gauge transformation for all fields but ϕ (see Table 1) and the Lagrangian must be invariant under it.

However, even though one can write down the terms in eqs. (18)-(21) in 3d, one cannot obtain such operators with a dimensional reduction computation from 4d with zero chemical potentials. This is because all these terms are T-odd and hence CPT-odd (in the 4d theory) according to Table 1. The statement that the P-violating operators without A_0 are CPT-odd is even more general: it can be seen from Table 1 that in the absence of A_0 , CT corresponds to complex conjugation so that any Hermitian (real) P-odd operator is odd in CPT. Thus for $\mu = 0$, finite temperature parity violation can only appear in the 3d effective theory of eq. (1) including A_0 .

3 Consequences of the P and C violating operators

Let us now consider some implications of the operators discussed. We have three comments to make.

1. The first implication concerns the general statement of dimensional reduction. According to the arguments in the Appendix, the following statement can be made:

Consider bosonic static n -point one-particle-irreducible (1PI) Matsubara Green's functions $G_n^{(4)}(\mathbf{p}_i)$ for the light ($m \sim g^2 T$) and heavy ($m \sim gT$) fields in the full 4d theory, depending on external 3-momenta \mathbf{p}_i . On the other hand, consider the corresponding 1PI Green's functions in the 3d theory of eq. (1). Multiply the 4d Green's functions by the factor $T^{n/2-1}$ to have the same dimension $\text{GeV}^{3-n/2}$ as the 3d Green's functions, and take a region of temperatures where $|m_3^2| \gtrsim \mathcal{O}(g^4 T^2)$. Then, there is a mapping of the temperature and the 4d coupling constants of the underlying theory to the 3d theory in eq. (1) such that the 3d theory gives the same light and heavy parity conserving Green's functions as the full 4d theory for $p \leq gT$ with a relative error at most of the order $\mathcal{O}(g^3)$,

$$\frac{\Delta G}{G} \lesssim \mathcal{O}(g^3). \quad (22)$$

The error arises from a powercounting estimate of the contributions of the neglected higher-dimensional operators to typical Green's functions inside the effective theory (see the Appendix). A similar conjecture was first made in Ref. [6] but with a more optimistic error estimate $\Delta G/G \sim \mathcal{O}(g^4)$. Let us here clarify a few points related to eq. (22).

First, note that near the critical point of the electroweak theory where the line of first order phase transition ends [22], the *relative* accuracy of Green's functions determination actually decreases because of the following obvious reason. Suppose that the parameters of the effective 3d theory are such that we are precisely at the critical point. Then the scalar mass, defined by the two-point Green's function, is exactly equal to zero. At the same time, there can be a mismatch in the parameters of the theory of the relative order g^3 , so that in the full 4d theory, the same Green's function need not be exactly zero for precisely the same Higgs mass and temperature.

In this sense, the relative accuracy $\mathcal{O}(g^3)$ is lost at this special point. Of course the endpoint itself exists also in the full 4d theory, but the corresponding Higgs mass value $m_{H,c}$ is displaced by a relative amount at most of order $\mathcal{O}(g^3)$.

Second, note that the statement concerning the accuracy of dimensional reduction in eq. (22) clearly does not apply to P-violating Green's functions. Indeed, these are exactly zero within the effective theory of eq. (1), yet non-zero in the full theory. The statement in eq. (22) only applies to P-even 1PI Green's functions which are non-zero in the effective theory.

It is important to stress that the Green's functions appearing in the conjecture of dimensional reduction are 1PI. For example, a correlation matrix employed in computing the masses of excitations coupling to each other, does not belong to this class. This case is discussed in more detail in point 2 below.

Finally, it should be noted that observables such as the critical temperature, the surface tension, or the latent heat, can be determined from P-even quantities and thus the super-renormalizable Lagrangian in eq. (1) is sufficient for all practical purposes. In particular, non-perturbative results on these observables such as in [22] remain intact by an addition of P-violating operators.

2. Consider the matrix of two-point correlators defined by the following commonly used operators:

$$\begin{aligned} H &= \phi^\dagger \phi, & (J^{PC} = 0^{++}) \\ W_i^3 &= i \left[(D_i \phi)^\dagger \phi - \phi^\dagger D_i \phi \right], & (J^{PC} = 1^{--}) \\ h_i &= \text{Tr } A_0 B_i. & (J^{PC} = 1^{++}) \end{aligned} \quad (23)$$

Note that by acquiring a non-zero momentum, the scalar state H can couple to an operator with quantum numbers $J^{PC} = 1^{-+}$.

Now, in the super-renormalizable theory, the quantum numbers P and C are conserved: all the operators in the Lagrangian are 0^{++} . This means that the states above cannot couple to each other, and their correlation matrix would only have diagonal components. However, in the full theory, there is P-violation, manifesting itself for instance through the existence of the 0^{--} -operator in eq. (11) which has a single A_0 . This means that states with $J^{PC} = 1^{++}$ and 1^{--} can couple to each other, and there are therefore non-zero cross-correlations between W_i^3 and h_i . The very long distance exponential fall-off of both operators is thus determined by a common mass, in contrast to what the super-renormalizable theory would suggest. This general problem has been discussed in [21]. Thus for this kind of Green's functions, the effects of P-violation are qualitatively important.

3. The third implication of the explicit parity violation induced by the higher order operators is related to the possibility of spontaneous parity violation. A priori, it is not excluded that the theory in eq. (1) exhibit spontaneous parity breaking in the symmetric phase [18]. However, lattice simulations in the symmetric phase of the

SU(2)+Higgs and SU(2) \times U(1)+Higgs models carried out in [19, 20] did not show any signal of spontaneous parity breaking. If such a phenomenon had taken place, then the small explicit P-violating operators in eqs. (3)-(5) induced by chemical potential could have altered the ground state of the electroweak theory significantly. Note that, in contrast, the operator in eq. (11) cannot have such an effect, as it is purely imaginary in Euclidian space [23].

4 Conclusions

We have discussed the role of parity violation at finite temperature. For most practical purposes, in particular for the determination of the thermodynamical properties of the electroweak phase transition, parity violation is not important, apart from changing the number of fermionic degrees of freedom participating in weak interactions. However, there are circumstances, such as the measurement of the Debye mass in the electroweak theory [21], where explicit parity violation does play a role.

Appendix

In this appendix we give the power counting argument behind the conjecture in eq. (22).

Let us first recall that slightly different formulations have been considered for dimensional reduction in the literature. The original one is a direct integration over the non-zero Matsubara modes (see, e.g., [3, 7]). However, at higher than 1-loop level, one has to be careful with this formulation, and, for instance, it turns out that in order to construct a local effective theory it is necessary to consider also the zero Matsubara modes in some internal loops in the reduction step [5, 6, 9]. The other formulation is based on matching [6, 8], and in this formulation the complication mentioned is automatically taken care of.

To now consider the accuracy of the matching procedure, let us assume the power counting rules $g_Y \sim g$, $\lambda \sim g^2$ characteristic of the renormalization structure of the theory. We also assume that the exact results of the full 4d theory could be derived from an effective 3d theory containing an infinite number of higher order operators, obtained by dimensional reduction. The fields of this effective theory are generically denoted by ϕ . We assume that renormalization is taken care of by the appropriate counterterms so that the only momentum scales in the effective theory are gT , g^2T . The question is now that if the 3d theory is truncated such that only the super-renormalizable Lagrangian in eq. (1) remains, what is the error induced for an arbitrary static Green's function? Note that to minimize the error, one is allowed to make an optimal choice for the values of the parameters remaining in the effective theory.

In general, the higher order operators generated by 1-loop dimensional reduction and

removed by the truncation, are in momentum space of the form

$$O_{4d} = g^n \frac{1}{T^{D-4}} \phi^n p^{D-n}, \quad (24)$$

where $D \geq 6$ is the dimension of the operator in 4d units, $2 \leq n \leq D$, and p is a generic momentum. If some operator is generated only at higher than 1-loop level, then the coupling constant appears in a still higher power. When one goes to 3d units, there is one extra $1/T$ multiplying the Lagrangian and at the same time the fields are scaled by $T^{1/2}$ and the couplings by its inverse, so that the operators in eq. (24) become

$$O_{3d} = g_3^n \frac{1}{T^{D-3}} \phi_3^n p^{D-n}. \quad (25)$$

We now study non-vanishing P-even 1PI Green's function inside the effective theory of eq. (1), and ask what kind of corrections arise from operators of the type in eq. (25). The relevance of the 1PI-condition is that all the loop momenta are of order gT, g^2T . Otherwise one could assign an arbitrarily small momentum to one of the lines.

One way of verifying eq. (22) is to apply Symanzik's conjecture [24] for effective field theories, which is widely used in the construction of improved lattice actions (see, e.g., [25]). In the present context, the conjecture can be viewed as basically a dimensional one and it becomes applicable when, after rescalings into the form in eq. (25), the inverse temperature $1/T$ is (dimensionally) identified with the lattice spacing a . It is then important to note that as has been discussed in the text (see also [15] for the P-even operators), the higher order operators start at $D = 6$ in the absence of chemical potentials. Then, the higher dimensional operators in eq. (25) are at most of order $\mathcal{O}(a^3)$. Adopting Symanzik's statement to our case, one then knows that in order to reproduce effects of order $\mathcal{O}(a^3)$, one should add to the super-renormalizable theory in eq. (1) all operators with $D = 6$, and modify the parameters of this theory (couplings, masses, and wave function normalizations), adding to the tree-level terms finite scaling corrections of order $\mathcal{O}(a^n)$ up to $n = 3$. Due to the fact that the effective 3d theory is super-renormalizable and contains dimensionful couplings only, the mapping can be done perturbatively by computing a finite number of multiloop diagrams (this is different in 4d at zero temperature, where the couplings are dimensionless and the mapping should be done in a non-perturbative way, see [25]). If this is done, then the errors remaining are $\mathcal{O}(a/\xi)^4 \sim \mathcal{O}(g^4)$, where the shortest physical correlation length ξ of the effective theory is of the order of the inverse Debye mass, $\xi \sim m_D^{-1}$.

If, on the other hand, the operators with $D = 6$ are *not included*, then the effective super-renormalizable theory can in principle have relative errors as large as $\mathcal{O}(a/\xi)^3 \sim \mathcal{O}(g^3)$, as stated in eq. (22). To be consistent with this precision, it is enough to compute the 3d parameters with the relative accuracy g^2 . For the coupling constants and wave function normalizations, one only needs a 1-loop computation, but for the scalar mass parameter one needs to go to 3-loop level [6] (or 2-loop level if the mass parameter is assumed to be of order $\sim g^2 T^2$).

Finally, let us note that the above arguments can be directly repeated for the “second step” of dimensional reduction, i.e., for the integration out of A_0 . One can see that the relative errors are $\mathcal{O}(g_3^2 a_0)^3 \sim \mathcal{O}(g^3)$ as stated in [6], where the cutoff scale $1/a_0$ is now identified with the Debye mass $m_D \sim gT$, and the only momentum scale in the final effective theory is $\sim g_3^2$. As mentioned above, P-violation is non-existent in the theory without A_0 , so that it does not cause any complications.

It is interesting to note that, in general, the relative accuracy obtained with a theory where A_0 is kept, is not better than with a theory from where A_0 is integrated out. Of course, the sets of Green’s functions considered are different. For those Green’s functions which are considered in the theory without A_0 , the theory with A_0 is in principle more accurate.

References

- [1] S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566; F. Wilczek and A. Zee, Phys. Rev. Lett. 43 (1979) 1571.
- [2] E. Witten, Nucl. Phys. B 223 (1983) 422.
- [3] P. Ginsparg, Nucl. Phys. B 170 (1980) 388; T. Appelquist and R. Pisarski, Phys. Rev. D 23 (1981) 2305; S. Nadkarni, Phys. Rev. D 27 (1983) 917.
- [4] K. Kajantie, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. B 407 (1993) 356.
- [5] K. Farakos, K. Kajantie, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. B 425 (1994) 67 [hep-ph/9404201].
- [6] K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. B 458 (1996) 90 [hep-ph/9508379].
- [7] A. Jakovác, K. Kajantie and A. Patkós, Phys. Rev. D 49 (1994) 6810; A. Jakovác and A. Patkós, Nucl. Phys. B 494 (1997) 54.
- [8] E. Braaten and A. Nieto, Phys. Rev. D 51 (1995) 6990; Phys. Rev. D 53 (1996) 3421.
- [9] A. Jakovác, Phys. Rev. D 53 (1996) 4538.
- [10] M. Laine, Nucl. Phys. B 481 (1996) 43; J.M. Cline and K. Kainulainen, Nucl. Phys. B 482 (1996) 73; McGill-97-7 [hep-ph/9705201]; M. Losada, Phys. Rev. D 56 (1997) 2893; G.R. Farrar and M. Losada, Phys. Lett. B 406 (1997) 60.
- [11] A. Rajantie, Nucl. Phys. B 501 (1997) 521.

- [12] M.E. Shaposhnikov, in *Proceedings of the Summer School on Effective Theories and Fundamental Interactions*, Erice, 1996 [CERN-TH-96-280, hep-ph/9610247].
- [13] S.Yu. Khlebnikov and M.E. Shaposhnikov, Phys. Lett. B 387 (1996) 817.
- [14] S. Chapman, Phys. Rev. C 47 (1993) 1763; Phys. Rev. D 50 (1994) 5308.
- [15] G.D. Moore, Phys. Rev. D 53 (1996) 5906.
- [16] A.I. Bochkarev, S.Yu. Khlebnikov and M.E. Shaposhnikov, Nucl. Phys. B 329 (1990) 493.
- [17] L. McLerran, M. Shaposhnikov, N. Turok and M. Voloshin, Phys. Lett. B 256 (1991) 451.
- [18] S.Yu. Khlebnikov and M.E. Shaposhnikov, Phys. Lett. B 254 (1991) 148.
- [19] J. Ambjørn, K. Farakos and M.E. Shaposhnikov, Mod. Phys. Lett. A 6 (1991) 3099; Nucl. Phys. B 393 (1993) 633.
- [20] K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. B 493 (1997) 413 [hep-lat/9612006].
- [21] P. Arnold and L.G. Yaffe, Phys. Rev. D 52 (1995) 7208.
- [22] K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. B 466 (1996) 189 [hep-lat/9510020]; Phys. Rev. Lett. 77 (1996) 2887.
- [23] C. Vafa and E. Witten, Phys. Rev. Lett. 53 (1984) 535.
- [24] K. Symanzik, Nucl. Phys. B 226 (1983) 187.
- [25] M. Lüscher, S. Sint, R. Sommer and P. Weisz, Nucl. Phys. B 478 (1996) 365.